Neutrino Mass via the Zee Mechanism in the 5D Split Fermion Model

Phys. Rev. D 83, 025017 (2011)

CS2011, Apr. 4th, 2011

We-Fu Chang, Yi-Ting Chen, and Siao-Cing Liou

Department of Physics, National Tsing Hua University, HsinChu 300, Taiwan

Outline

Introduction

Goal

Zee Model (FCNC, Parameters)

Extra dimensions & split fermions

Zee Model + 5-dim Split Fermion Model

—Reduce Free Parameters

—FCNC Suppressed

Model Setup

Neutrinos get Majorana masses from Zee model in 5D

Numerical Result

Phenomenology

Conclusions

Introduction

Goal

Neutrino oscillation → neutrino masses

Zee model: a 2-higgs doublets model

FCNC problem

A. Zee. *Physics Letters B*, 93(4):389 – 393, 1980.

Extra-dimensions

mass hierarchy: wavefunction overlapping

FCNC problem: BC for higgs in extra Dim

Introduction

Zee model

A. Zee. *Physics Letters B*, 93(4):389 – 393, 1980.

neutrino masses through radiative correction.

Give the neutrino Majorana mass by introducing a new higgs singlet h^+ and a new higgs doublet Φ_2 in SM.

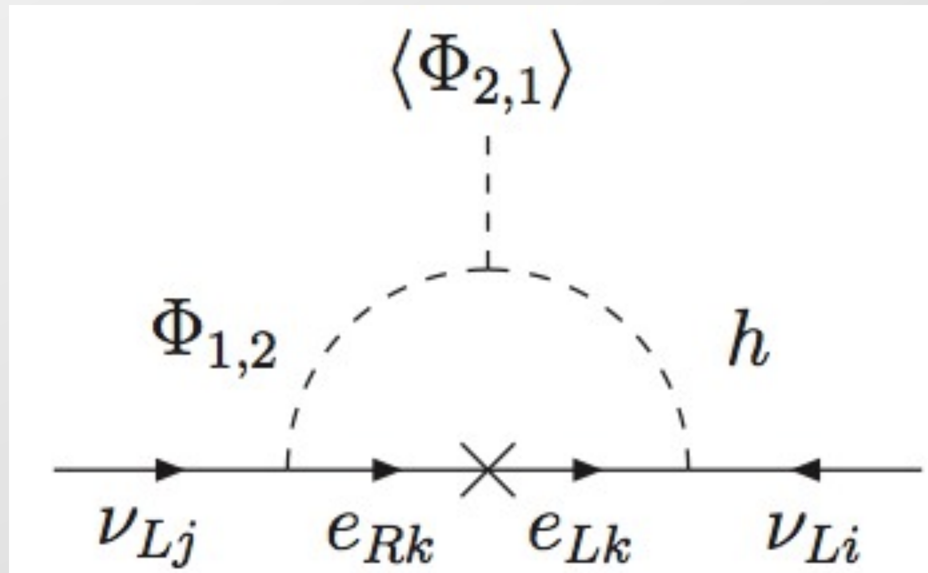
$$\mathcal{L}_{Zee} = -f^1_{ab}\bar{\Psi}_{aL}\Phi_1 e_{bR} - f^2_{ab}\bar{\Psi}_{aL}\Phi_2 e_{bR} - M_{12}\Phi_1 i\tau_2\Phi_2 h^* - f^h_{ab}\overline{\Psi^{c}_{aL}} i\tau_2\Psi_{bL} h + H.c.$$

		QEDcharge	SU(2)L charge	hyper charge
		Q	I_3	$\frac{Y}{2}$
new singlet	h	$+1$	0	$+1$
higgs doublet	$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}$	$\begin{pmatrix} +1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{2}$
higgs doublet	$\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$	$\begin{pmatrix} +1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{2}$
lepton doublet-L	$\Psi_L = \begin{pmatrix} \nu_L \\ l_L^- \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ \frac{-1}{2} \end{pmatrix}$	$-\frac{1}{2}$
charge lepton-R	l_R^-	-1	0	-1

Introduction

Zee model

$$\mathcal{L}_{Zee} = -f^{1}_{ab}\bar{\Psi}_{aL}\Phi_1 e_{bR} - f^{2}_{ab}\bar{\Psi}_{aL}\Phi_2 e_{bR} - M_{12}\Phi_1 i\tau_2\Phi_2 h^* - f^{h}_{ab}\overline{\Psi^{\mathbf{c}}_{aL}} i\tau_2\Psi_{bL} h + H.c.$$



Neutrino mass matrix (Majorana)

9+9+3

too many parameters(21 complex Yukawa couplings...)

FCNC problems(Two higgs doublets)

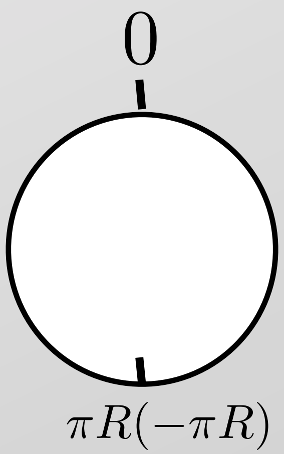
A. Zee. *Physics Letters B*, 93(4):389 – 393, 1980.

Introduction

Extra dimensions

The 5th extra dimensions are compactified in a very small regions, like circles with radii R, so the Lagrangian and fields are modified.

❶ Lagrangian in (4+1) dimensions:



$$\mathcal{S} = \int d^4x\, dy\, \mathcal{L}_{(4+1)D}$$

thus

$$\mathcal{L}_{4D} = \int dy\, \mathcal{L}_{(4+1)D}$$

❷ Kaluza-Klein(KK) decomposition :
the extra dimensional part of a field can be expanded in a complete set $f_n(y)$

$$\overset{(5)}{\Phi}(x^\mu, y) = \sum_n \phi^n(x^\mu) f_n(y)$$

We can choose a orthonormal basis:

$$\int f_n^*(y) f_m(y) dy = \delta_{nm}$$

Introduction

Extra dimensions

The 5th extra dimensions are compactified in a very small regions, like circles with radii R, so the Lagrangian and fields are modified.

❷ Kaluza-Klein(KK) Particles:

example:

$$\mathcal{L}^{4D} = \int dy \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi + \frac{1}{2}\partial_y \Phi \partial^y \Phi - \frac{1}{2} m^2 \Phi^2$$

$$\Phi(x^\mu, y) = \sum_n \phi^n(x^\mu) \underline{f_n(y)} \sim cos(\frac{n}{R}y)$$

$$\mathcal{L}^{4D} = \sum_n \frac{1}{2}\partial_\mu \phi^n \partial^\mu \phi^n - \underbrace{\frac{1}{2}(\frac{n}{R})^2(\phi^n)^2 - \frac{1}{2}m^2(\phi^n)^2}_{\text{kk mass}}$$

Introduction

Extra dimensions

The 5th extra dimensions are compactified in a very small regions, like circles with radii R, so the Lagrangian and fields are modified.

❷ Kaluza-Klein(KK) Particles:

ex $$\mathcal{L}^{4D} = \sum_n \frac{1}{2}\partial_\mu \phi^n \partial^\mu \phi^n - \frac{1}{2}(\frac{n}{R})^2(\phi^n)^2 - \frac{1}{2}m^2(\phi^n)^2$$

n	m_n^2
$\vdots$	$\vdots$
4	$\frac{16}{R^2} + m^2$
3	$\frac{9}{R^2} + m^2$
2	$\frac{4}{R^2} + m^2$
1	$\frac{1}{R^2} + m^2$
0	m^2

Introduction

Split Fermions

Nima Arkani-Hamed* and Martin Schmaltz, PHYSICAL REVIEW D, VOLUME 61, 033005 (2000)

- Background field: $\Phi(y) = \pm 2\mu^2 y$
- KK zero mode, localized in 5th D
- zero mode chiral fermions, massless! in 4D
  (separate L & R)

$$\mathcal{L}_{5D} = \overline{\Psi}^{(5)}(x^\mu, y)(i\Gamma^\mu \partial_\mu + i\Gamma^5 \partial_y - \underbrace{\Phi(y)}_{\text{background field}})\Psi^{(5)}(x^\mu, y)$$

Introduction

Split Fermions

Nima Arkani-Hamed* and Martin Schmaltz,
PHYSICAL REVIEW D, VOLUME 61, 033005 (2000)

- Background field: $\Phi(y) = \pm 2\mu^2 y$
- KK zero mode, localized in 5th D
- zero mode chiral fermions, massless! in 4D → identify as SM fermions in 4D

(separate L & R)

$$\mathcal{L}_{5D} = \overline{\Psi}^{(5)}(x^\mu, y)(i\Gamma^\mu\partial_\mu + i\Gamma^5\partial_y - \underbrace{\Phi(y)}_{\text{background field}})\Psi^{(5)}(x^\mu, y)$$

Introduction

Split Fermions

Nima Arkani-Hamed* and Martin Schmaltz,
PHYSICAL REVIEW D, VOLUME 61, 033005 (2000)

- Background field: $\Phi(y) = \pm 2\mu^2 y$
- KK zero mode, localized in 5th D
- zero mode chiral fermions, massless! in 4D → identify as SM fermions in 4D

$$\mathcal{L}_{5D} = \overline{\Psi}^{(5)}(x^\mu, y)(i\Gamma^\mu \partial_\mu + i\Gamma^5 \partial_y - \underbrace{\Phi(y)}_{\text{background field}})\Psi^{(5)}(x^\mu, y)$$

$$\Gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad , \quad \Gamma^5 = i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Gamma matrices in 5D

5D Dirac equation

$$(i\Gamma^\mu \partial_\mu + i\Gamma^5 \partial_y - \Phi(y))\Psi^{(5)}(x^\mu, y) = 0$$

KK expansion

$$\Psi^{(5)}(x^\mu, y) = \begin{pmatrix} \sum_{n=0}^{\infty} \chi_n(x^\mu) g_n(y) \\ \sum_{n=0}^{\infty} \eta_n(x^\mu) f_n(y) \end{pmatrix}$$

Introduction

Split Fermions

Solutions

back ground field $\Phi(y) = \pm 2\mu^2 y$

$$\Psi_0(x^\mu) = \begin{pmatrix} \chi_0(x^\mu) \\ \eta_0(x^\mu) \end{pmatrix} \quad g_0(y) = f_0(y) = \underline{\frac{\mu^{1/2}}{(\pi/2)^{1/4}} e^{-\mu^2 y^2}} \quad m_n = 2\mu\sqrt{n}$$

Gaussian packet in 5th D

$\Phi(y) = 2\mu^2 y$

$$\mathcal{L}_{4D} = \int dy \mathcal{L}_{5D} \supset \underline{i\overline{\psi_{0L}}\gamma^\mu \partial_\mu \psi_{0L}}$$

Left zero mode, massless in 4D!

Identify as SM Left-handed leptons

$\Phi(y) = -2\mu^2 y$

$$\mathcal{L}_{4D} = \int dy \mathcal{L}_{5D} \supset \underline{i\overline{\psi_{0R}}\gamma^\mu \partial_\mu \psi_{0R}}$$

Right zero mode, massless in 4D!

Identify as SM Right-handed leptons

Introduction

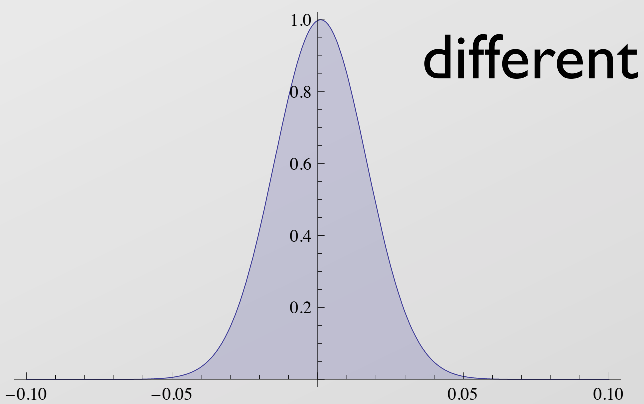
Split Fermions

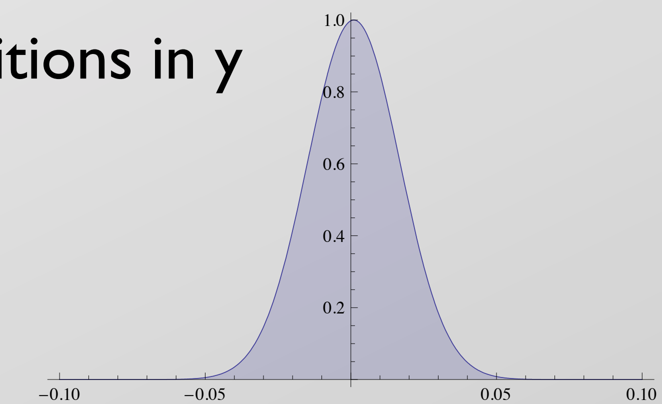
Solutions

back ground field $\Phi(y) = \pm 2\mu^2 y$

$$\Psi_0(x^\mu) = \begin{pmatrix} \chi_0(x^\mu) \\ \eta_0(x^\mu) \end{pmatrix} \qquad g_0(y) = f_0(y) = \underbrace{\frac{\mu^{1/2}}{(\pi/2)^{1/4}} e^{-\mu^2 y^2}}_{\text{Gaussian packet in 5th D}} \qquad m_n = 2\mu\sqrt{n}$$

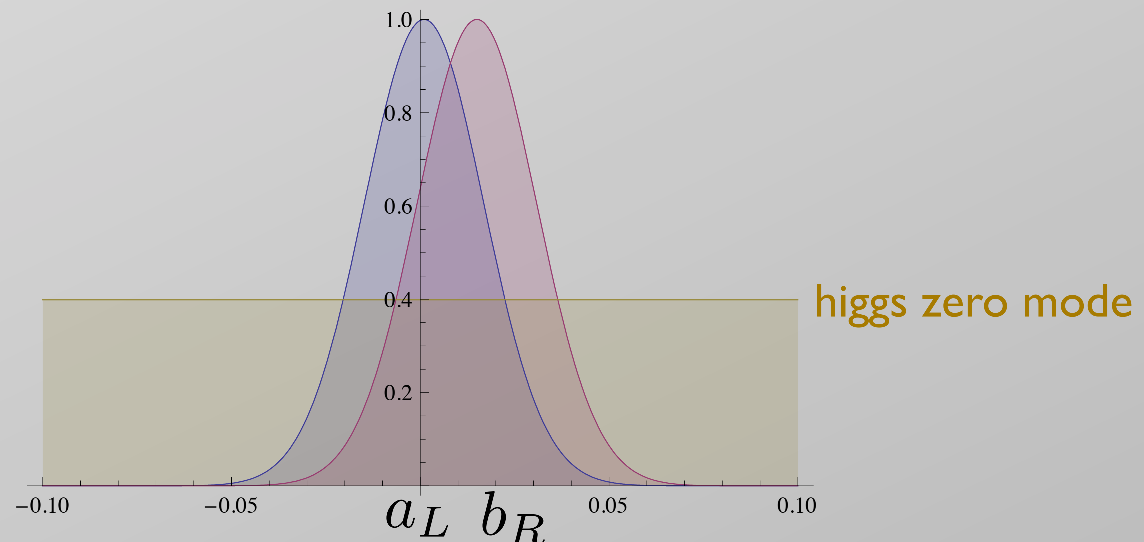
different positions in y





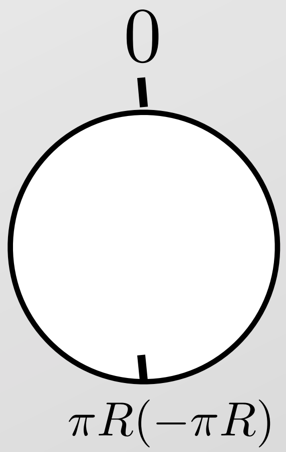
$$y \longrightarrow (y - a_L) \qquad\qquad y \longrightarrow (y - b_R)$$

Yukawa: mass hierarchy in 4D



Introduction

Zee model + Split Fermion in 5D



our model

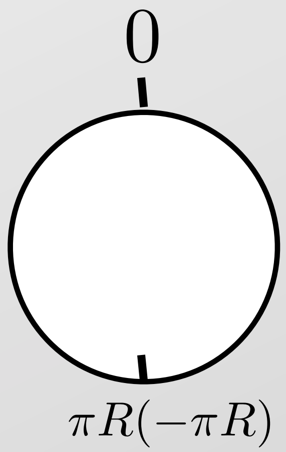
mass hierarchy: wavefunction overlapping

S^1/Z_2

FCNC problem: assign BC for Higgs in extra Dim.

Introduction

Zee model + Split Fermion in 5D

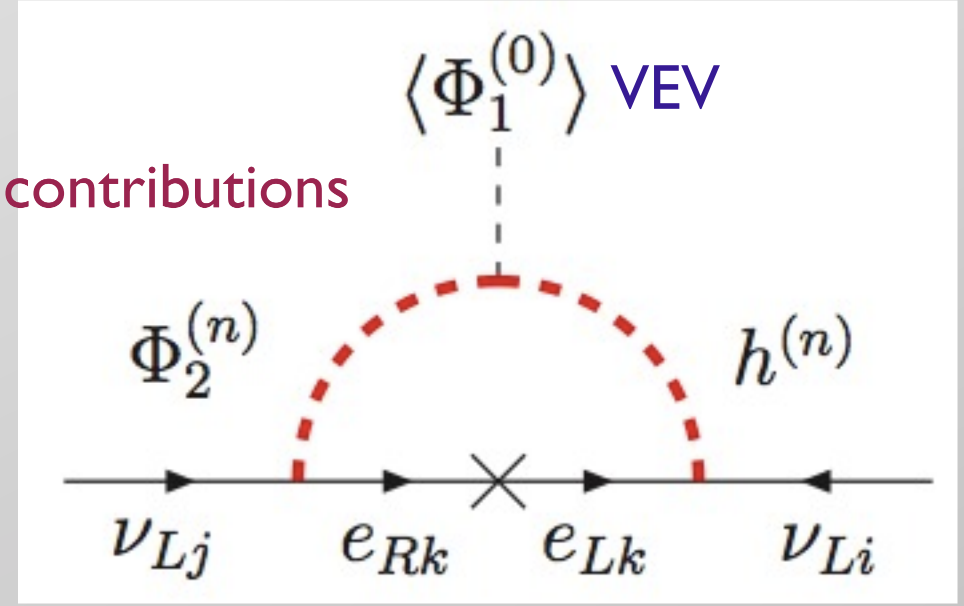


- Zee Model + Compactified-5D Split-fermion:
  11 parameters and order 1 Yukawa couplings in 5D

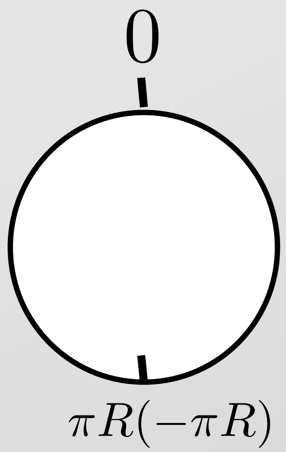
$$
\begin{aligned}
\mathcal{L}_{5DZee} = & -\sqrt{2\pi R}\hat{f}^{1}_{ab}\overline{\hat{\Psi}_{aL}}\hat{\Phi}_1\hat{e}_{bR} - \sqrt{2\pi R}\hat{f}^{2}_{ab}\overline{\hat{\Psi}_{aL}}\hat{\Phi}_2\hat{e}_{bR} \\
& -\sqrt{2\pi R}\hat{f}^{h}_{ab}\overline{\hat{\Psi}^{c}_{aL}}i\tau_2\hat{\Psi}_{bL}\hat{h} - \frac{\kappa}{\sqrt{2\pi R}}\hat{\Phi}_1 i\tau_2\hat{\Phi}_2\hat{h}^* + H.c.\,,
\end{aligned}
$$

$$\xrightarrow[\text{effective 4-D}]{\int_{-\pi R}^{\pi R} dy} \quad \sum_n$$

kk contributions



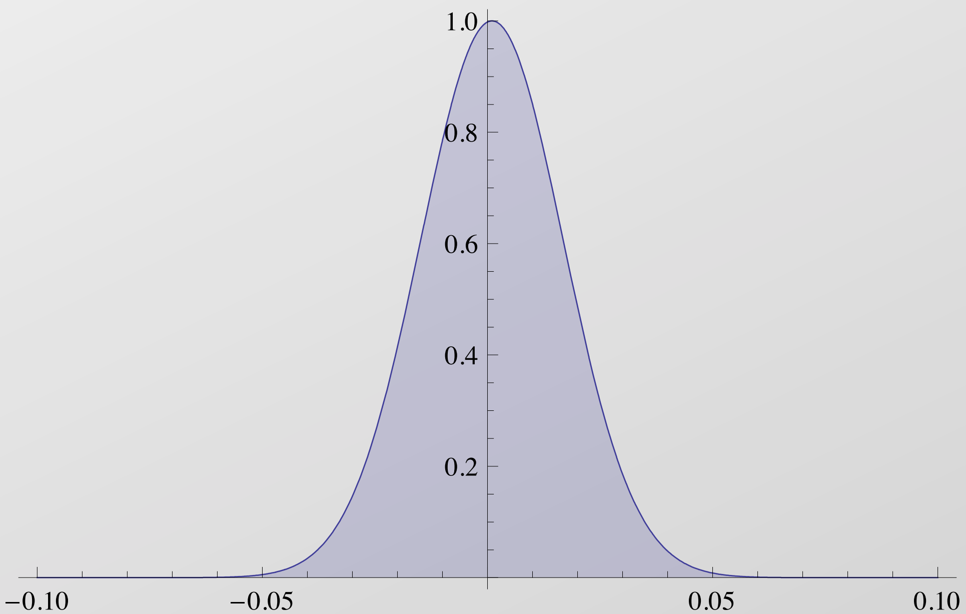
Introduction
Zee model + Split Fermion in 5D



- Zee Model + Compactified-5D Split-fermion:
  11 parameters and order 1 Yukawa couplings in 5D

3+3 Leptons in the 5th-dim

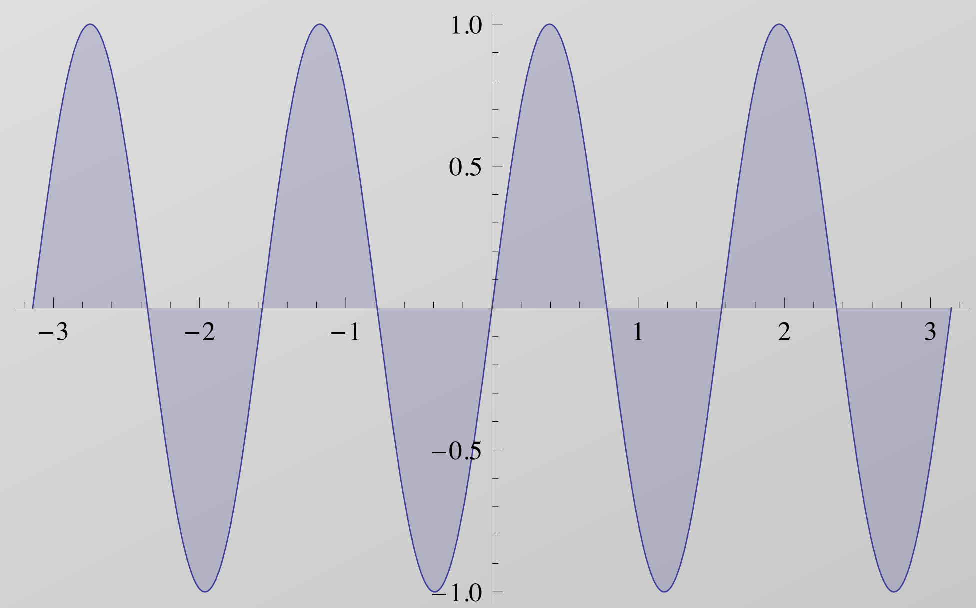
near the origin



Gaussian

$$\{c_1^R, c_2^R, c_3^R, c_1^L, c_2^L, c_3^L\}$$

2+1 Higgs in the 5th-dim



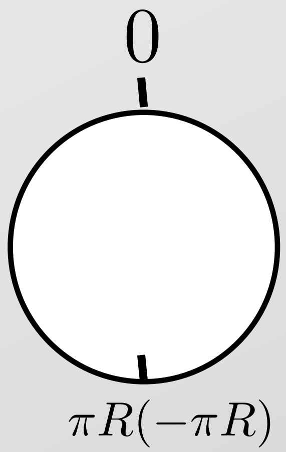
Sin or Cos

$\hat{h}$ $\hat{\Phi}_2$ $\hat{\Phi}_1$

Introduction

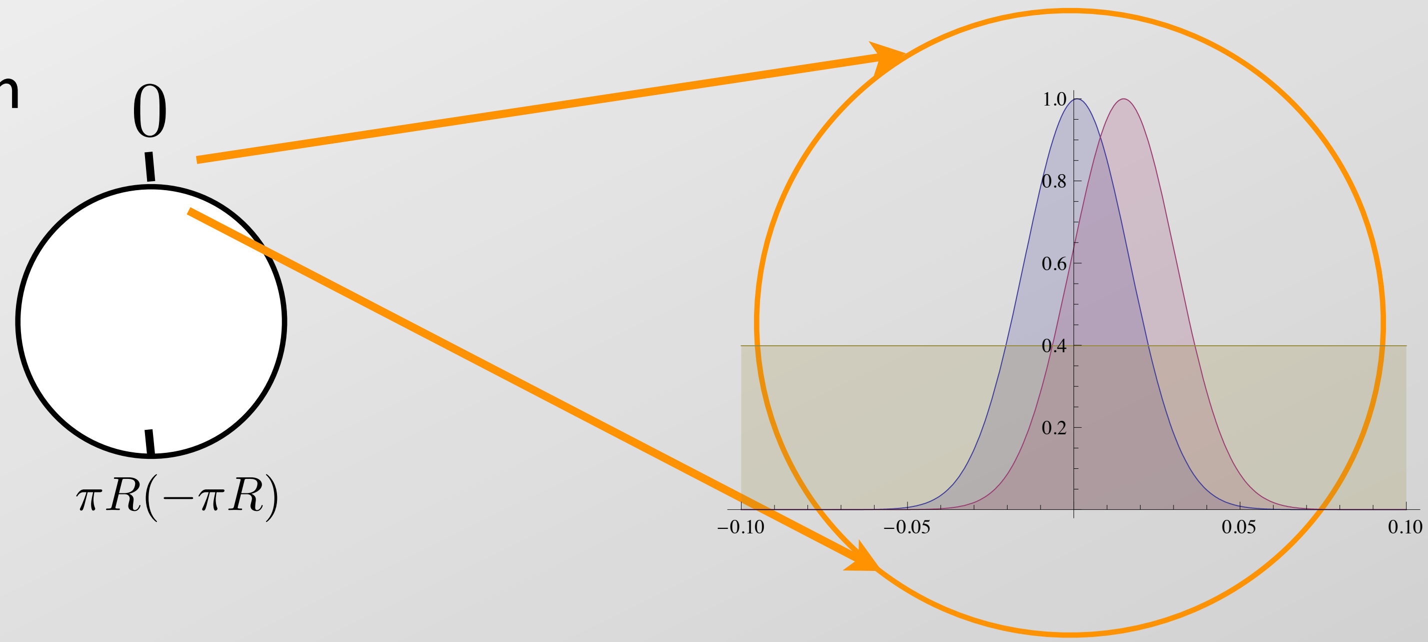
Zee model + Split Fermion in 5D

Charge lepton masses



$\hat{\Phi}_1$ Even parity in 5th -> SM Higgs

5th-dim



Split fermion & SM Higgs :
Gaussian distribution, constant & overlapping

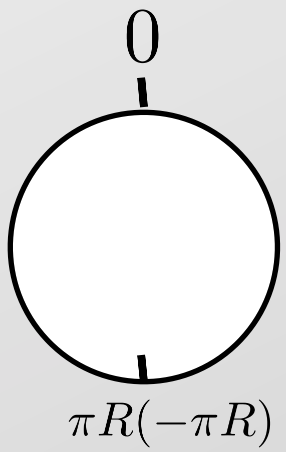
Effective 4D

$$\xrightarrow{\int_{-\pi R}^{\pi R} dy}$$

charge lepton mass hierarchy

Introduction

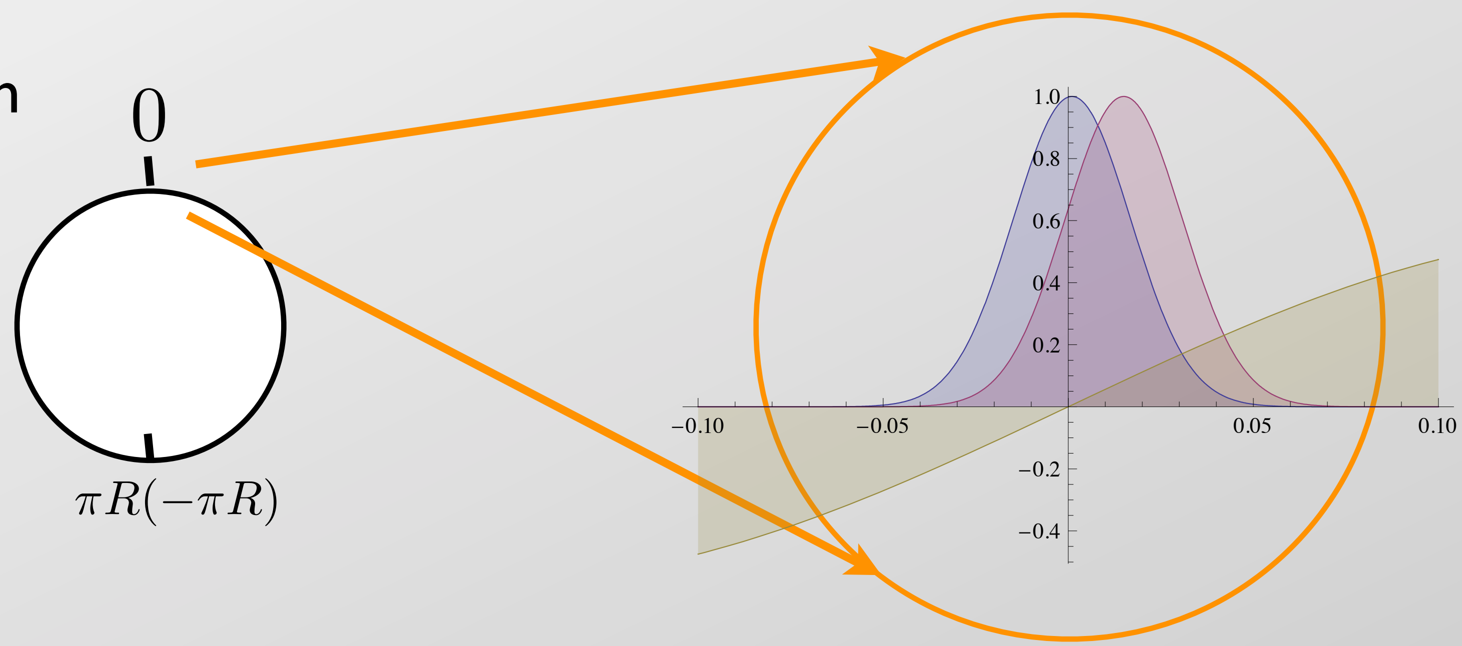
Zee model + Split Fermion in 5D



Neutrino masses

No VEV $\hat{h}$ $\hat{\Phi}_2$ Odd parity in 5th

5th-dim



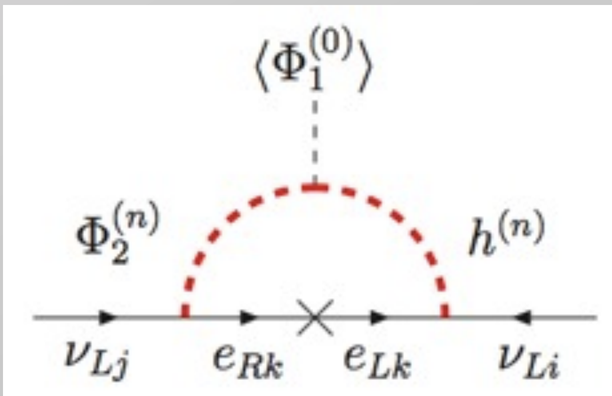
Split fermion & 2 Extra higgs :
Gaussian distribution, Sin(y) & overlapping

Effective 4D

$$\int_{-\pi R}^{\pi R} dy \longrightarrow$$

small Yukawa coupling

Small neutrino masses

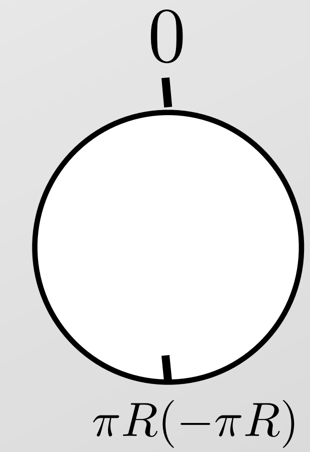


Introduction

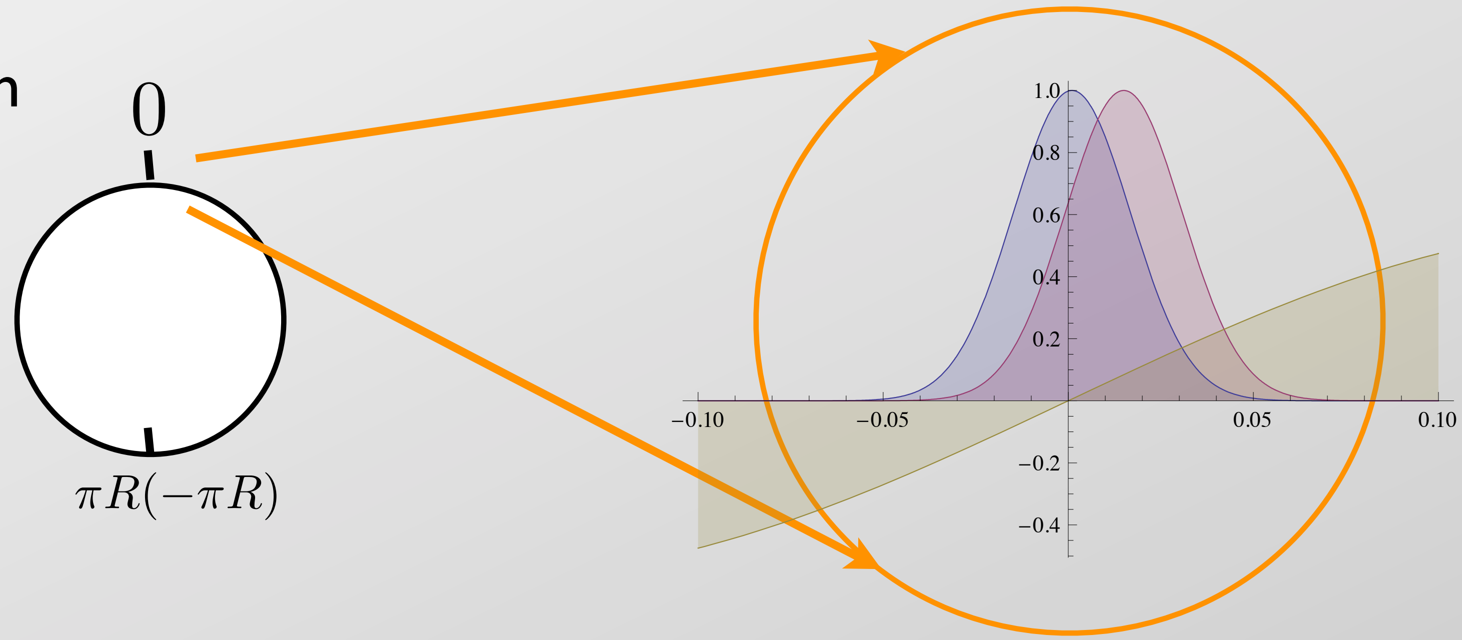
Zee model + Split Fermion in 5D

Neutrino masses

No VEV $\hat{h}$ $\hat{\Phi}_2$ Odd parity in 5th



5th-dim

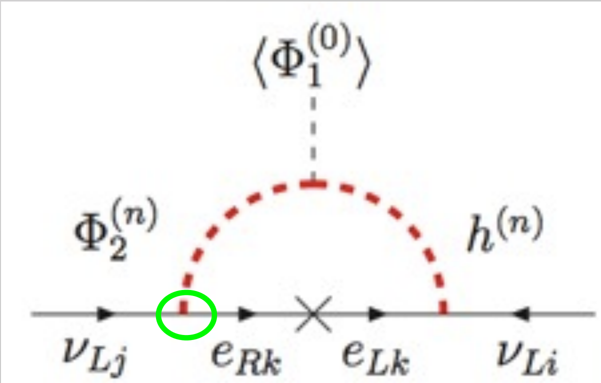


Split fermion & 2 Extra higgs :
Gaussian distribution, Sin(y) & overlapping

Effective 4D

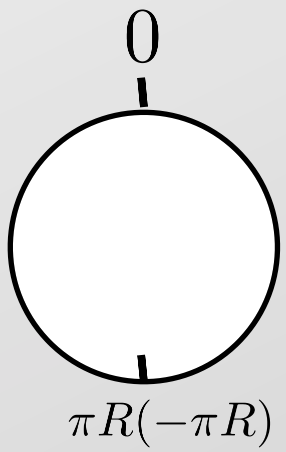
small Yukawa coupling

$$\xrightarrow{\int_{-\pi R}^{\pi R} dy}$$ Small neutrino masses



Introduction

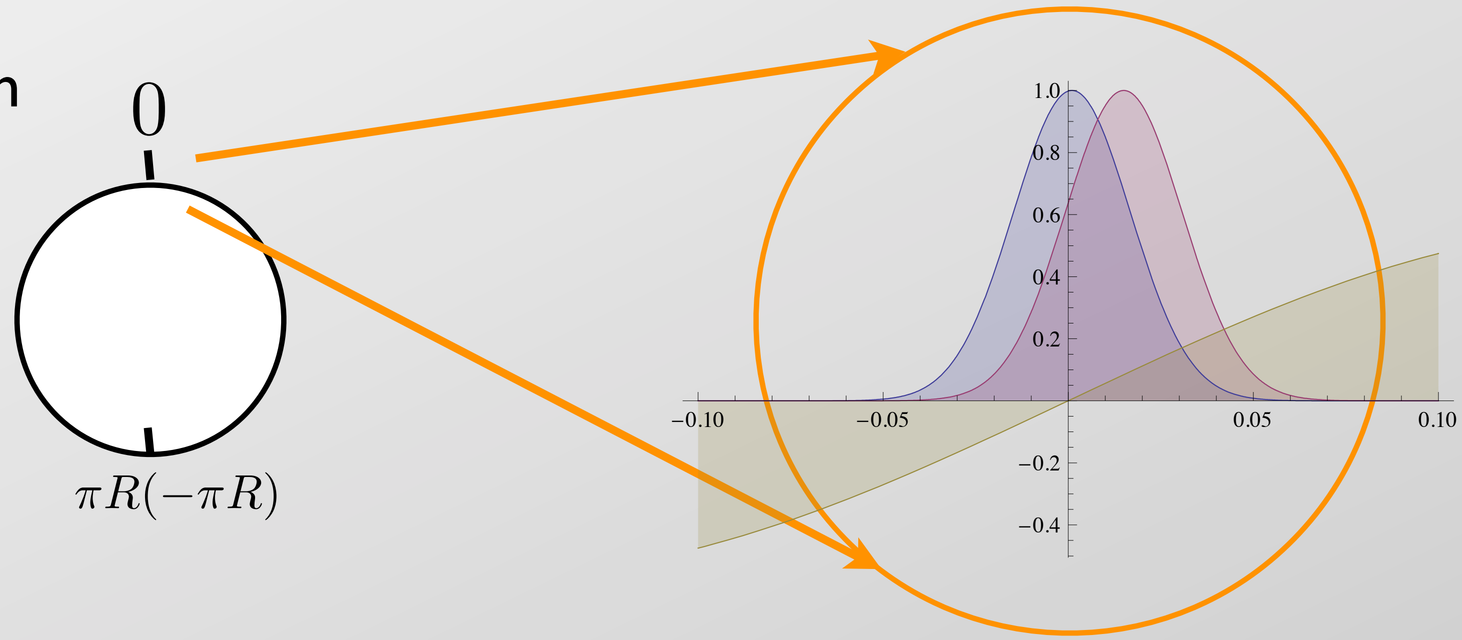
Zee model + Split Fermion in 5D



Neutrino masses

No VEV $\hat{h}$ $\hat{\Phi}_2$ Odd parity in 5th

5th-dim



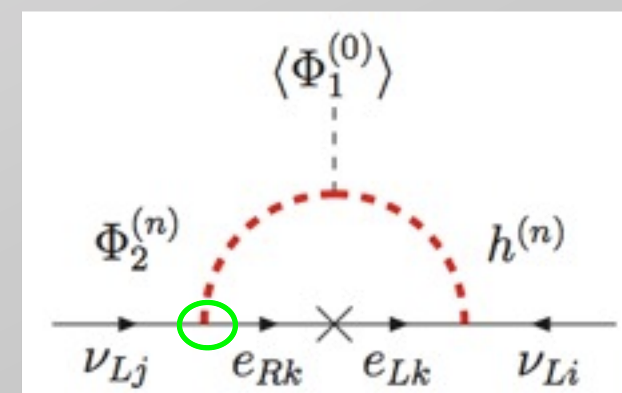
Split fermion & 2 Extra higgs :
Gaussian distribution, Sin(y) & overlapping

$$\int_{-\pi R}^{\pi R} dy \longrightarrow$$

Effective 4D

small Yukawa coupling

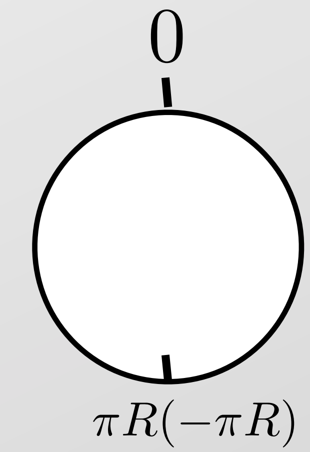
Small neutrino masses



$$f_{ab}^{2(n)}$$

Introduction

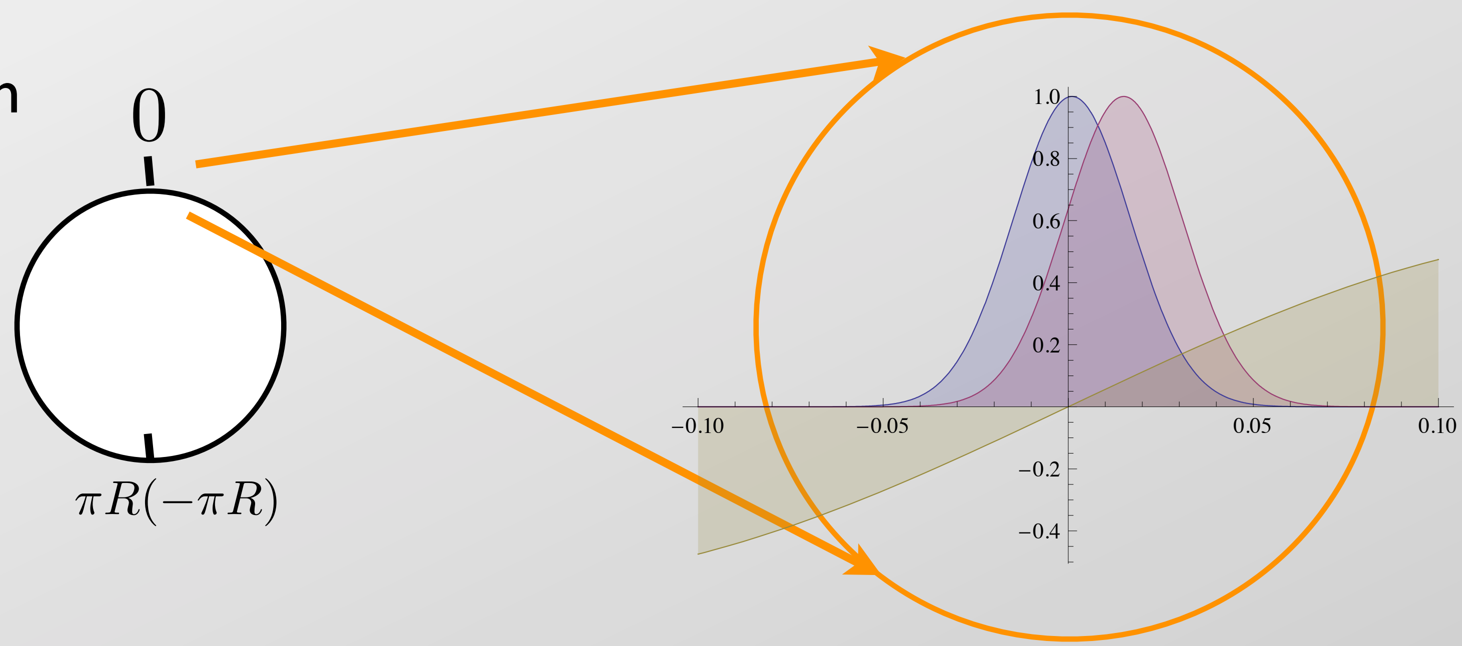
Zee model + Split Fermion in 5D



Neutrino masses

No VEV $\hat{h}$ $\hat{\Phi}_2$ Odd parity in 5th

5th-dim



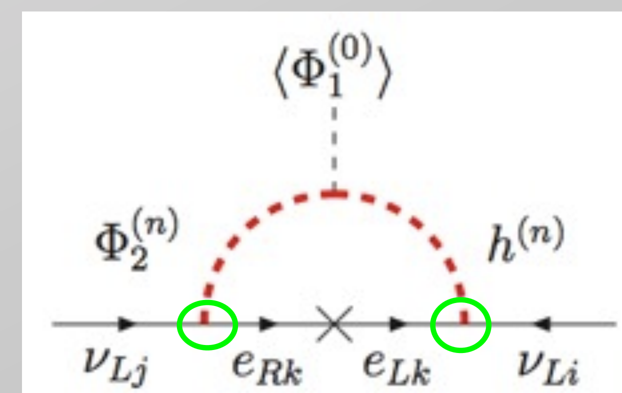
Split fermion & 2 Extra higgs :
Gaussian distribution, Sin(y) & overlapping

$$\int_{-\pi R}^{\pi R} dy \longrightarrow$$

Effective 4D

small Yukawa coupling

Small neutrino masses

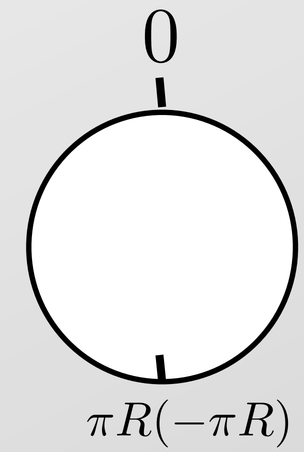


$$f_{ab}^{2(n)}$$

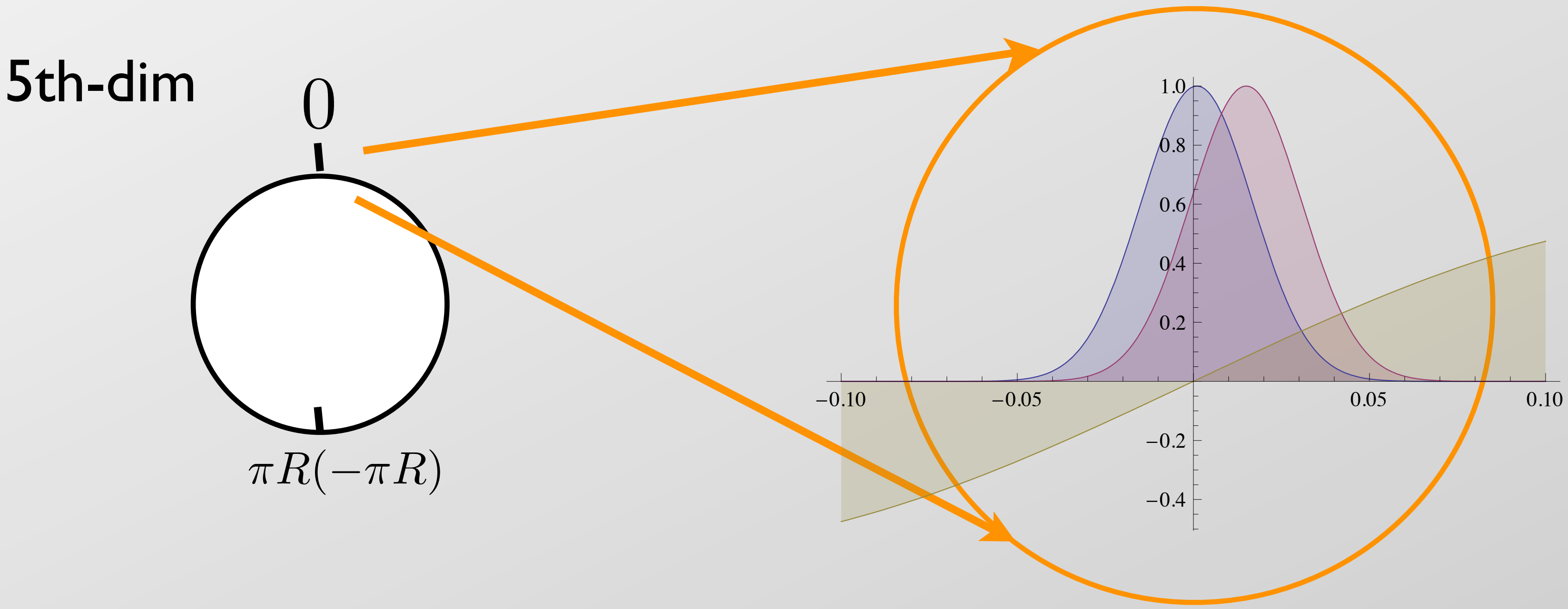
Introduction

Zee model + Split Fermion in 5D

Neutrino masses



No VEV $\hat{h}$ $\hat{\Phi}_2$ Odd parity in 5th



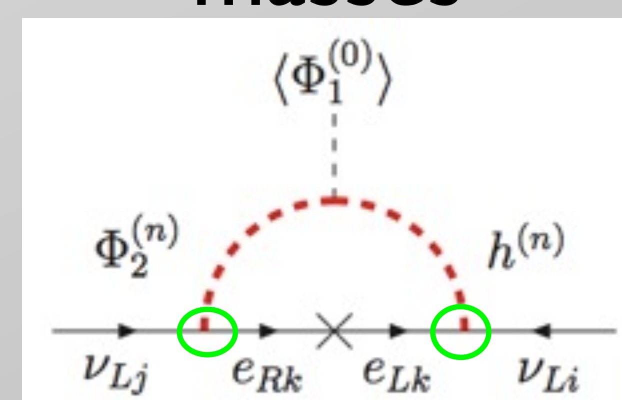
Split fermion & 2 Extra higgs :
Gaussian distribution, Sin(y) & overlapping

$$\int_{-\pi R}^{\pi R} dy \longrightarrow$$

Effective 4D

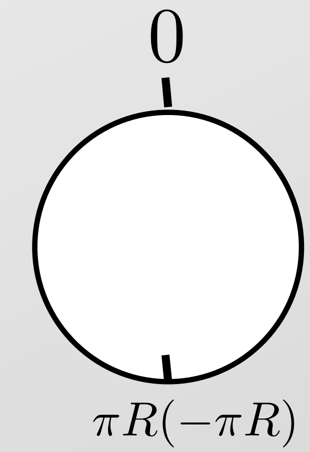
small Yukawa coupling

Small neutrino masses



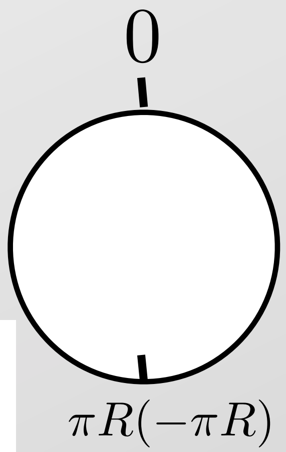
$f_{ab}^{2(n)}$ $f_{ab}^{h(n)}$

Model Setup



Zee model in 5 dimensions

Model Setup



Zee model in 5 dimensions

$$
\begin{aligned}
\mathcal{L}_{5DZee} &= -\sqrt{2\pi R}\hat{f}^{1}_{ab}\overline{\hat{\Psi}}_{aL}\hat{\Phi}_1\hat{e}_{bR} - \sqrt{2\pi R}\hat{f}^{2}_{ab}\overline{\hat{\Psi}}_{aL}\hat{\Phi}_2\hat{e}_{bR} \\
&\quad -\sqrt{2\pi R}\hat{f}^{h}_{ab}\overline{\hat{\Psi}^{c}_{aL}}i\tau_2\hat{\Psi}_{bL}\hat{h} - \frac{\kappa}{\sqrt{2\pi R}}\hat{\Phi}_1 i\tau_2\hat{\Phi}_2\hat{h}^{*} + H.c.\,,
\end{aligned}
$$

❶ Leptons in 5D:

$$
\hat{\psi}_a(x^\mu, y) \supset \underline{\psi_{a0L}(x^\mu)} \times \frac{\sqrt{\mu}}{\left(\frac{\pi}{2}\right)^{\frac{1}{4}}} e^{-\mu^2(y-\underline{c^L_a})^2}
$$

SM L-lepton doublets in 4D

$$
\hat{l}_a(x^\mu, y) \supset \underline{l_{a0R}(x^\mu)} \times \frac{\sqrt{\mu}}{\left(\frac{\pi}{2}\right)^{\frac{1}{4}}} e^{-\mu^2(y-\underline{c^R_a})^2}
$$

SM R-charged-lepton singlets in 4D

❷ kk Higgs:

SM higgs in 4D

$$
\hat{\Phi}_1(x, y) = \underline{\frac{1}{\sqrt{2\pi R}}\Phi^{(0)}_1(x)} + \frac{1}{\sqrt{\pi R}}\sum_{n=1}\cos\frac{ny}{R}\Phi^{(n)}_1(x) \quad \text{(even in y)}
$$

$$
\hat{\Phi}_2(x, y) = \frac{1}{\sqrt{\pi R}}\sum_{n=1}\sin\frac{ny}{R}\Phi^{(n)}_2(x), \quad \text{(odd in y)}
$$

$$
\hat{h}(x, y) = \frac{1}{\sqrt{\pi R}}\sum_{n=1}\sin\frac{ny}{R}h^{(n)}(x). \quad \text{(odd in y)}
$$

Model Setup

4D Effective

Effective 4D zee model with kk particles:

$$\boxed{\int_{-\pi R}^{\pi R} dy}$$

❶ $$f_{ab}^{1(n)} \simeq (\sqrt{2})^{1-\delta_{n,0}}\,\hat{f}_{ab}^{1} \exp\left[\frac{-(c_a^L - c_b^R)^2}{2\sigma^2}\right] \cos\frac{n(c_a^L + c_b^R)}{2R}\,,$$

❷ $$f_{ab}^{2(n)} \simeq \sqrt{2}\,\hat{f}_{ab}^{2} \exp\left[\frac{-(c_a^L - c_b^R)^2}{2\sigma^2}\right] \sin\frac{n(c_a^L + c_b^R)}{2R}\,,$$

❸ $$f_{ab}^{h(n)} \simeq \sqrt{2}\,\hat{f}_{ab}^{h} \exp\left[\frac{-(c_a^L - c_b^L)^2}{2\sigma^2}\right] \sin\frac{n(c_a^L + c_b^L)}{2R}\,,$$

$$\mathcal{L}_{Zee}^{New} \supset -\frac{\kappa}{2\pi R}\sum_{n,m=1}^{\infty} \delta_{n,m}\,\Phi_1^{(0)} i\tau_2 \Phi_2^{(n)} h^{(m)*} + H.c.\,,$$

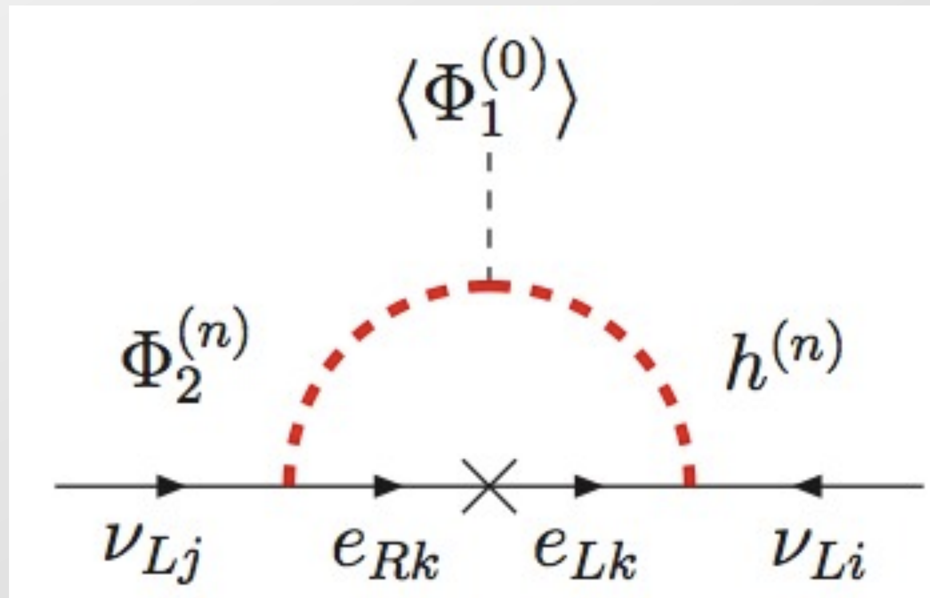
higgs cubic term

$$\mathcal{M}_{ab}^{e} = \hat{f}_{ab}^{1}\frac{v}{\sqrt{2}} \exp\left[\frac{-(c_a^L - c_b^R)^2}{2\sigma^2}\right] \longleftarrow$$

charge lepton mass matrix

Model Setup

Neutrino mass



kk contributions

$$\mathcal{M}^{\nu}_{ij} \sim \frac{1}{16\pi^2} \sum_{n=1}^{\infty} \sum_{k=e,\mu,\tau} \left(\frac{v}{\sqrt{2}}\right) \left(\frac{\kappa}{2\pi R}\right) \frac{m_k \left(f_{ik}^{2(n)}\right)^* f_{kj}^{h(n)}}{M^2_{\Phi_2,n} - M^2_{h,n}} \ln \frac{M^2_{\Phi_2,n}}{M^2_{h,n}} + (i \leftrightarrow j)$$

Neutrino mass matrix

Numerical Result

Numerical Result

parameters:

μ, R, $M_{h,0}$ $M_{\Phi_2,0}$, κ $\{c_1^R, c_2^R, c_3^R, c_1^L, c_2^L, c_3^L\}$

❶ Choose the parameters:

$$\mu = 2 \times 10^3 \text{ TeV}, \quad R^{-1} = 1\text{TeV}$$

$$M_{h,0} = 400 \text{ GeV and } M_{\Phi_2,0} = 200 \text{ GeV}$$

❷ $\hat{f}_{ab} = \rho_{ab} e^{i\theta_{ab}}$ (5D Yukawa couplings: no hierarchy) ρ_{ab}

Let their components to have relative random coefficients in $[0.5 \sim 1.5]$
and random phases in $[0 \sim 2\pi]$ θ_{ab}

❸ Search $\{c_1^R, c_2^R, c_3^R, c_1^L, c_2^L, c_3^L\}$, range: [0, 20] (1/μ), and κ

Numerical Result

4 sets are all Inverted hierarchy.

Configuration	κ (3-higgs coupling)	c_1^R	c_2^R	c_3^R	c_1^L	c_2^L	c_3^L
I	0.389	10.112	2.989	9.592	14.350	13.954	6.060
II	1.054	9.789	9.570	10.557	5.715	13.498	5.201
III	0.169	9.416	8.956	18.602	5.881	13.249	13.591
IV	0.974	1.371	8.159	17.663	12.595	12.106	4.346

TABLE I: The four viable configurations which can accommodate charge lepton and neutrino data in the same time. The split fermion location c's are in the unit of $\sigma(= 5 \times 10^{-4} R)$.

Configuration	m_e(MeV)	m_μ(MeV)	m_τ(GeV)	$\sin^2(2\theta_{12})$	$\sin^2(2\theta_{23})$	θ_{13} (rad)
I	3.1 ± 1.5	120(22)	1.73(31)	0.79(24)	0.43(26)	0.11(8)
II	6.3 ± 3.0	119(20)	2.49(48)	0.84(18)	0.72(24)	0.16(11)
III	0.64(12)	122(22)	1.70(31)	0.76(27)	0.56(27)	0.33(20)
IV	0.49(10)	78(14)	2.25(43)	0.83(20)	0.93(08)	0.13(7)
experiment:	0.51	105	1.776	0.87	> 0.92	< 0.19

TABLE II: Charged lepton masses and neutrino mixings in the 4 viable configurations

Particle Data Group

Numerical Result

neutrinoless double beta decay

4 sets are all
Inverted hierarchy.

Configuration	m_1^ν	m_2^ν	m_3^ν	$\lvert m_{ee}^\nu \rvert$
I	38 ± 13	46 ± 14	1.4 ± 1.3	14 ± 7
II	41 ± 16	45 ± 15	5.1 ± 4.2	6 ± 3
III	40 ± 16	45 ± 16	6.2 ± 5.0	8 ± 4
IV	39 ± 16	49 ± 15	5 ± 7	9 ± 5

TABLE III: The absolute neutrino masses and the effective neutrino mass $\lvert m_{ee}^\nu \rvert$ (in meV).

$$\lvert m_{ee}^\nu \rvert \sim 0.01 eV$$

Decay mode	Conf. I	Conf. II	Conf. III	Conf. IV	experiment:
$Br(\mu^- \to e^+e^-e^-)$	$4(2) \times 10^{-13}$	$1.6(6) \times 10^{-13}$	$2(1) \times 10^{-13}$	$1.3(7) \times 10^{-13}$	$< 10^{-12}$
$Br(\tau^- \to e^+e^-e^-)$	$1.9(9) \times 10^{-11}$	$9(6) \times 10^{-14}$	$1.5(1.5) \times 10^{-14}$	$1.3(1.3) \times 10^{-18}$	$< 3.6 \times 10^{-8}$
$Br(\tau^- \to \mu^+\mu^-e^-)$	$1.0(5) \times 10^{-11}$	$5(3) \times 10^{-14}$	$1.0(9) \times 10^{-14}$	$1.2(1.2) \times 10^{-18}$	$< 3.7 \times 10^{-8}$
$Br(\tau^- \to e^+e^-\mu^-)$	$4(3) \times 10^{-13}$	$3.0(2.8) \times 10^{-14}$	$2.8(2.6) \times 10^{-13}$	$3(2) \times 10^{-13}$	$< 2.7 \times 10^{-8}$
$Br(\tau^- \to \mu^+\mu^-\mu^-)$	$7(6) \times 10^{-13}$	$5.3(5.0) \times 10^{-14}$	$7(6) \times 10^{-13}$	$1.1(6) \times 10^{-12}$	$< 3.2 \times 10^{-8}$

TABLE IV: Lepton flavor violating decays

Particle Data Group

We-Fu Chang and John N. Ng,, Phys. Rev. D 71, 053003 (2005)

Phenomenology

FCNC suppressed

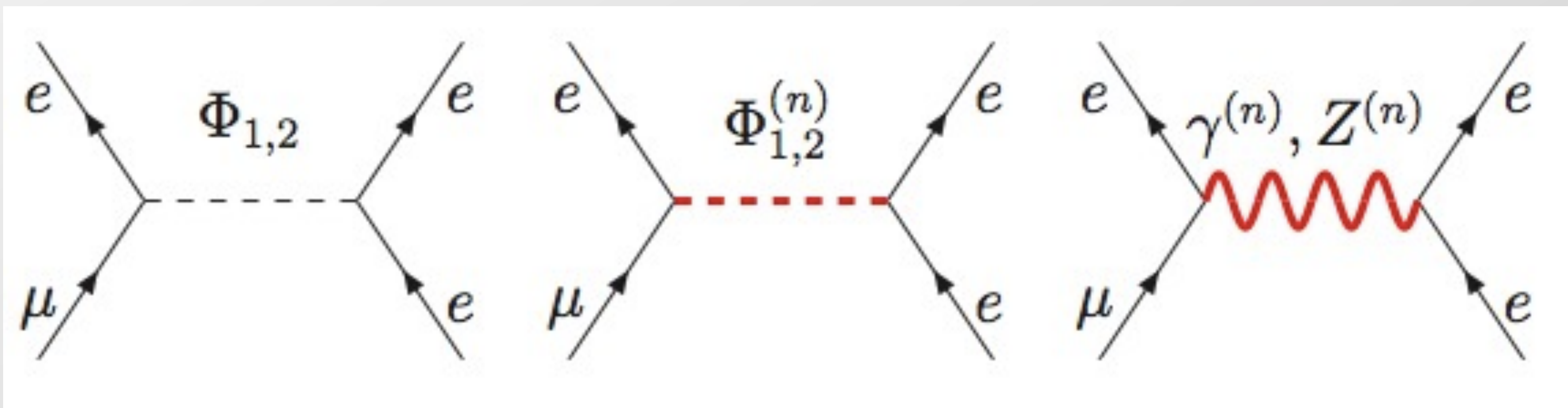
Phenomenology

FCNC suppressed

$ex\!: \ \mu \rightarrow 3e$

Original Zee Model

Zee Model in 5D with kk modes



$$\frac{\mathcal{M}^{\gamma}_{5D}}{\mathcal{M}^{\Phi_2}_{5D}} \sim \frac{e^2\{U^\dagger_{L/R}[\cos\frac{\sigma}{R}]U_{L/R}\}_{\mu e}}{\frac{m_e}{v}\{U^\dagger_{L/R}[\sin\frac{\sigma}{R}]U_{R/L}\}_{\mu e}} \sim \frac{4\pi\alpha}{(m_e m_\mu/v^2)}\frac{(\sigma/R)^2}{(\sigma/R)} \sim 10^4$$

$$\mathrm{diag}\,\{m_e, m_\mu, m_\tau\} = U^\dagger_L \mathcal{M}^e U_R$$

$$\sigma = \frac{1}{\mu}$$

width of the split fermion

$$\frac{\mathcal{M}^{\gamma}_{5D}}{\mathcal{M}^{H}_{4D}} \sim \frac{4\pi\alpha\{U^\dagger_{L/R}\frac{\sigma^2}{R^2}U_{L/R}\}_{\mu e}/(1/R)^2}{\{U^\dagger_{L/R}\frac{m_\mu}{v}U_{R/L}\}_{\mu e}(m_e/v)/M_H^2} \sim 10^{-1}$$

Conclusions

•Instead of 21 complex parameters in Zee model, our model has 11 plus order 1 Yukawa couplings with random phases.

•In this model, the idea of split fermion transform the mass hierarchy for leptons into the position-differences in the 5th dimension.

•Smallness of neutrino mass & FCNC suppressed: the wavefunction overlap & BC for Higgs in Extra-Dim.

•Through roughly estimating, the ranges of the parameters cover the experimental data of charge lepton masses, the three mixing angles in PMNS matrix, and the differences of neutrino mass squares.

•we found a inverted mass hierarchy solution for neutrinos.

•Predictions of nonzero $\theta_{13}, |m^{\nu}_{ee}| \sim 0.01eV$

End

Thank you for listening!

Appendex

Split Fermions

Solutions

back ground field $\Phi(y) = \pm 2\mu^2 y$

$$\Psi_0(x^\mu) = \begin{pmatrix} \chi_0(x^\mu) \\ \eta_0(x^\mu) \end{pmatrix} \quad g_0(y) = f_0(y) = \frac{\mu^{1/2}}{(\pi/2)^{1/4}} e^{-\mu^2 y^2} \quad m_n = 2\mu\sqrt{n}$$

Gaussian packet in 5th D

$\Phi(y) = 2\mu^2 y$

$$\mathcal{L}_{4D} = \int dy \mathcal{L}_{5D} = \underline{i\overline{\psi_{0L}}\gamma^\mu \partial_\mu \psi_{0L}} + \sum_{n=1}^{\infty} i\overline{\tilde{\psi}_n}\gamma^\mu \partial_\mu \tilde{\psi}_n - m_n \overline{\tilde{\psi}_n}\tilde{\psi}_n$$

Left zero mode, massless in 4D!

$\Phi(y) = -2\mu^2 y$

Identify as SM Left-handed leptons

$$\mathcal{L}_{4D} = \int dy \mathcal{L}_{5D} = \underline{i\overline{\psi_{0R}}\gamma^\mu \partial_\mu \psi_{0R}} + \sum_{n=1}^{\infty} i\overline{\tilde{\psi}'_n}\gamma^\mu \partial_\mu \tilde{\psi}'_n + m_n \overline{\tilde{\psi}'_n}\tilde{\psi}'_n$$

Right zero mode, massless in 4D!

Identify as SM Right-handed leptons